AIAA JOURNAL
Vol. 30, No. 7, July 1992

Readers' Forum

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Compatibility of Measured and Predicted Vibration Modes in Model Improvement Studies"

Irving U. Ojalvo*
Columbia University, New York, New York 10027

THE present writer offers some comments on the main issue of the subject paper[1] and suggests a possible improvement to treatment of that paper's final equation.

The subject paper has as its main point that there is generally an incompatibility between the analytical and experimental modal information for dynamic systems of interest. Therefore, correlation studies that seek to localize errors so as to improve the math model must either first 1) condense the analytical model or 2) expand the test data.

The present writer suggests that a third alternative is possible and at times superior to options 1 and 2 just cited; namely, to neither condense the analytical system degrees of freedom nor expand the test data base using the (perhaps less accurate) analytical model. This alternative simply compares available test data with its corresponding analytically determined values to determine the modeling corrections. In this way, neither the reduced analytical formulation nor the expanded modal test vectors become biased or contaminated. In addition, there is no requirement with this procedure that each test mode provide data for the same degrees of freedom. This latter condition could occur, for example, if a particular accelerometer gave reliable data for some modes but not for others.

A modal improvement/localization procedure that use this third alternative of modal comparison was first proposed by Collins et al.[2] and was later developed and implemented by Ojalvo et al.[3]

In this procedure, one uses a Taylor expansion of the less-detailed modal test data $\{\phi_T\}$ and corresponding truncated analytical data $\{\bar{\phi}_A\}$ to obtain

$$\{\phi_T\} = \{\bar{\phi}_A\} + [S]\{\Delta r\} + \{R\} \tag{1}$$

where

$$[S] = \left[\frac{\partial\{\bar{\phi}_A\}}{\partial\{r\}}\right] \tag{2}$$

where $\{\Delta r\}$ are the analytical model error localization or parameter corrections to improve correlation, and $\{R\}$ are the corresponding residual terms of the Taylor series plus experimental errors.

A least-squares minimization of the residuals in the weighted form

$$\{R\}^T[W]\{R\}$$

then leads to the system equations

$$[S]^T[W][S]\{\Delta r\} = [S]^T[W](\{\phi_T\} - \{\bar{\phi}_A\}) \tag{3}$$

which may be solved for $\{\Delta r\}$ by ordinary decomposition of $[S]^T[W][S]$ if it is nonsingular or by epsilon decomposition[4] if it is singular.

The second point that the present writer addresses concerns the subject paper's Eq. (10), in which the matrix product $[\phi_c][\phi_c]^T$ is rank deficient. Taking the transpose of Eq. (10) in Ref. 1 leads to

$$[\phi_c][\phi_c]^T([\Delta K] + i[H]) = [\phi_c][\cdot\,\omega_c^2\,\cdot][\phi_c]^T[M_c] - [\phi_c][\phi_c]^T[K_a] \tag{4}$$

This equation may be solved for $([\Delta K] + i[H])$ via singular value decomposition (SVD).[5] The resulting solution, although not unique, will yield the "minimum norm" solution for $([\Delta K] + i[H])$, i.e., the solution for which $[K_c]$ is closest to $[K_a]$ and is often physically most justifiable.

The SVD solution for Eq. (4) is

$$([\Delta K] + i[H]) = X\Lambda^{-1}X^T(\phi_c\omega_c^2\phi_c^TM_c - \phi_c\phi_c^TK_a) \tag{5}$$

where Λ are the nonzero eigenvalues of $[\phi_c]$ and $[\phi_c]^T$ and X are its corresponding eigenvectors.

A final minor point is made here regarding the subject paper's Fig. 2, in which the square shaded areas represented in $[K_a]_{12}$ and $[K_a]_{21}$ should be interchanged.

References

[1]He, J., and Ewins, D. J., "Compatibility of Measured and Predicted Vibration Modes in Model Improvement Studies," *AIAA Journal,* Vol. 29, No. 5, 1991, pp. 798–803.

[2]Collins, J. D., Hart, G. C., Hasselman, T. K., and Kennedy, B., "Statistical Identification of Structures," *AIAA Journal,* Vol. 12, No. 2, 1974, pp. 185–190.

[3]Ojalvo, I. U., Ting, T., and Pilon, D., "PAREDYM—A Parameter Refinement Computer Code for Structural Dynamic Models," *International Journal of Analytical and Experimental Modal Analysis,* Vol. 5, No. 1, 1989, pp. 43–49.

[4]Ojalvo, I. U., and Ting, T., "Interpretation and Improved Solution Approach for Ill-Conditioned Linear Equations," *AIAA Journal,* Vol. 28, No. 11, 1990, pp. 1976–1979.

[5]Maia, N. M. M., "An Introduction to the Singular Value Decomposition Technique (SVD)," *Proceedings of the 7th International Modal Analysis Conference,* Vol. 1, Union College, Schenectady, NY, and Society for Experimental Mechanics, Bethel, CT, 1990, pp. 335–339.

Received July 1, 1991; revision received Oct. 9, 1991; accepted for publication Oct. 21, 1991.

*Senior Research Scientist, Department of Civil Enginering and Engineering Mechanics, 610 Seeley W. Mudd Building.

Reply by the Authors to I. U. Ojalvo

J. He* and D. J. Ewins†
Imperial College, London SW7-2BX, England, United Kingdom

THE authors thank I. U. Ojalvo for his interest in and comments on their recent paper[1] and wish to affirm and expand on the points made in his Comment.

Received Oct. 1, 1991; accepted for publication Oct. 9, 1991.

*Research Student; currently Lecturer, Footscray Inst. of Technology, Melbourne, Australia.

†Professor, Department of Mechanical Engineering, Exhibition Road.